and by a process entirely analogous to that commencing with equation (9), we find

$$\cos\theta_2 = \frac{\sqrt{(a^2-c^2)}}{a} + \left(\frac{a-\sqrt{(a^2-c^2)}}{a}\right)\left(1-\frac{10c^2}{7a^2}\right)^4 \quad \ldots\ldots(16),$$

and generally, by inference,

$$\cos\theta_r = \frac{\sqrt{(a^2-c^2)}}{a} + \left(\frac{a-\sqrt{(a^2-c^2)}}{a}\right)\left(1-\frac{10c^2}{7a^2}\right)^{2r} \quad \ldots\ldots(17).$$

I. Solution by the PROPOSER.

Let C be the center of the sphere at the instant of impact, ω_{r-1}, ω_r, the angular velocities of the sphere just before and just after the rth impact, respectively; $\angle ACB=2\beta$, k^2=radius of gyration, then $(a^2+k^2)\omega_r = (k^2+a^2\cos 2\beta)\omega_{r-1}$. Now, $\sin\beta=c/a$, $k^2=\frac{2}{5}a^2$.

$$\therefore \tfrac{7}{5}a^2\omega_r = a^2\left(\tfrac{7}{5}-\frac{2c^2}{a^2}\right)\omega_{r-1}. \quad \therefore \frac{\omega_r}{\omega_{r-1}} = 1-\frac{10c^2}{7a^2}.$$

$\therefore 1-\dfrac{10c^2}{7a^2}$ must always be positive. Let ω=angular velocity just before first impact. By the principle of energy,

$$\tfrac{1}{2}(a^2+k^2)\omega^2 = ga(1-\cos\beta),$$
$$\tfrac{1}{2}(a^2+k^2)\omega_r^2 = ga(\cos\theta_r-\cos\beta).$$

$$\therefore \frac{\omega_r^2}{\omega^2} = \frac{\cos\theta_r-\cos\beta}{1-\cos\beta}. \quad \cos\theta_r = \cos\beta + (1-\cos\beta)\left(\frac{\omega_r}{\omega}\right)^2.$$

$$\cos\beta = \frac{\sqrt{(a^2-c^2)}}{a}, \qquad \frac{\omega_r}{\omega} = \left(1-\frac{10c^2}{a^2}\right)^r.$$

$$\therefore \cos\theta_r = \frac{\sqrt{(a^2-c^2)}}{a} + \frac{a-\sqrt{(a^2-c^2)}}{a}\left(1-\frac{10c^2}{a^2}\right)^{2r}.$$

PROBLEMS FOR SOLUTION.

ALGEBRA.

247. Proposed by PROFESSOR G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Find the sum, to n terms, of

$$1+\frac{n}{2}+\frac{n(n+2)}{2.4}+\frac{n(n+2)(n+4)}{2.4.6}+\ldots\ldots$$

248. Proposed by S. A. COREY. Hiteman, Iowa.

Prove that $\frac{6435}{2}\cdot\frac{161280^2}{929569}\left[1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\right]=\pi^{16}$.

249. Proposed by J. J. KEYES, Fogg High School, Nashville, Tenn.

Solve $x+y+z=5$, $x^2+y^2=z^2$, $x^3+y^3+z^3=8$.

AVERAGE AND PROBABILITY.

173. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A chord of length c is drawn at random in a given ellipse. What is the average area of the segment cut off by the chord?

CALCULUS.

209. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A thread makes $n(=30)$ equidistant spiral turns around a rough cone whose altitude is $h(=10$ feet), and radius of base $r(=11$ inches). How far will a bird fly in unwinding the thread if the part unwound is at all times perpendicular to the axis of the cone?

210. Proposed by EDWIN L. RICH, Schenectady, New York.

Determine a polynomial, $f(x)$, entirely in x and of the seventh degree, so that $f(x)+1$ is divisible by $(x-1)^4$ and $f(x)-1$ by $(x+1)^4$. [Goursat-Hedrick, *A Course in Mathematical Analysis*, p. 32, Ex. 3.]

211. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

If $x=v^{1/(v-1)}$, what is the $f(x)$ such that $v=f(x)$?

DIOPHANTINE ANALYSIS.

130. Proposed by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Stroud, England.

In how many ways may a number of which the prime factors are known, be expressed as the sum of two or more consecutive numbers?

131. Proposed by DR. O. E. GLENN. Drury College.

Devise a method of finding the cubic residues of a number, analogous to Gauss's "Method of Exclusion" for quadratic residues.

GEOMETRY.

274. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

If a straight line AB is placed between two intersecting straight lines MN and PQ and is made to revolve through all possible positions having A always in MN and B always in PQ, what is the locus of any point L in AB or AB produced?